Polynomials - Scientific Notation

Objective: Multiply and divide expressions using scientific notation and exponent properties.

One application of exponent properties comes from scientific notation. Scientific notation is used to represent really large or really small numbers. An example of really large numbers would be the distance that light travels in a year in miles. An example of really small numbers would be the mass of a single hydrogen atom in grams. Doing basic operations such as multiplication and division with these numbers would normally be very combersome. However, our exponent properties make this process much simpler.

First we will take a look at what scientific notation is. Scientific notation has two parts, a number between one and ten (it can be equal to one, but not ten), and that number multiplied by ten to some exponent.

$$\textbf{Scientific Notation: } a \times 10^b \textbf{ where } 1 \leqslant a < 10$$

The exponent, b, is very important to how we convert between scientific notation and normal numbers, or standard notation. The exponent tells us how many times we will multiply by 10. Multiplying by 10 in affect moves the decimal point one place. So the exponent will tell us how many times the exponent moves between scientific notation and standard notation. To decide which direction to move the decimal (left or right) we simply need to remember that positive exponents mean in standard notation we have a big number (bigger than ten) and negative exponents mean in standard notation we have a small number (less than one).

Keeping this in mind, we can easily make conversions between standard notation and scientific notation.

Example 1.

Convert $14,200$ to scientific notation	Put decimal after first nonzero number
1.42	Exponent is how many times decimal moved, 4
$\times 10^4$	Positive exponent, standard notation is big
1.42×10^4	Our Solution

Example 2.

Convert 0.0042 to scientific notation	Put decimal after first nonzero number
4.2	Exponent is how many times decimal moved, 3
$\times 10^{-3}$	Negative exponent, standard notation is small
4.2×10^{-3}	Our Solution

Example 3.

Convert 3.21×10^5 to standard notation	Positive exponent means standard notation big number. Move decimal right 5 places
$321,000$	Our Solution

Example 4.

Conver 7.4×10^{-3} to standard notation	Negative exponent means standard notation is a small number. Move decimal left 3 places
0.0074	Our Solution

Converting between standard notation and scientific notation is important to understand how scientific notation works and what it does. Here our main interest is to be able to multiply and divide numbers in scientific notation using exponent properties. The way we do this is first do the operation with the front number (multiply or divide) then use exponent properties to simplify the 10's. Scientific notation is the only time where it will be allowed to have negative exponents in our final solution. The negative exponent simply informs us that we are dealing with small numbers. Consider the following examples.

Example 5.

$(2.1 \times 10^{-7})(3.7 \times 10^5)$	Deal with numbers and $10's$ separately
$(2.1)(3.7) = 7.77$	Multiply numbers
$10^{-7}10^5 = 10^{-2}$	Use product rule on $10's$ and add exponents
7.77×10^{-2}	Our Solution

Example 6.

$\dfrac{4.96 \times 10^4}{3.1 \times 10^{-3}}$	Deal with numbers and $10's$ separately
$\dfrac{4.96}{3.1} = 1.6$	Divide Numbers
$\dfrac{10^4}{10^{-3}} = 10^7$	Use quotient rule to subtract exponents, be careful with negatives!
	Be careful with negatives, $4 - (-3) = 4 + 3 = 7$
1.6×10^7	Our Solution

Example 7.

$(1.8\times10^{-4})^3$	Use power rule to deal with numbers and $10's$ separately
$1.8^3=5.832$	Evaluate 1.8^3
$(10^{-4})^3=10^{-12}$	Multiply exponents
5.832×10^{-12}	Our Solution

Often when we multiply or divide in scientific notation the end result is not in scientific notation. We will then have to convert the front number into scientific notation and then combine the 10's using the product property of exponents and adding the exponents. This is shown in the following examples.

Example 8.

$(4.7\times10^{-3})(6.1\times10^{9})$	Deal with numbers and $10's$ separately
$(4.7)(6.1)=28.67$	Multiply numbers
2.867×10^{1}	Convert this number into scientific notation
$\mathbf{10^1}10^{-3}10^{9}=10^{7}$	Use product rule, add exponents, using 10^1 from conversion
2.867×10^{7}	Our Solution

World View Note: Archimedes (287 BC - 212 BC), the Greek mathematician, developed a system for representing large numbers using a system very similar to scientific notation. He used his system to calculate the number of grains of sand it would take to fill the universe. His conclusion was 10^{63} grains of sand because he figured the universe to have a diameter of 10^{14} stadia or about 2 light years.

Example 9.

$\dfrac{2.014\times10^{-3}}{3.8\times10^{-7}}$	Deal with numbers and $10's$ separately
$\dfrac{2.014}{3.8}=0.53$	Divide numbers
$0.53=5.3\times10^{-1}$	Change this number into scientific notation
$\dfrac{\mathbf{10^{-1}}10^{-3}}{10^{-7}}=10^{3}$	Use product and quotient rule, using 10^{-1} from the conversion
	Be careful with signs:
	$(-1)+(-3)-(-7)=(-1)+(-3)+7=3$
5.3×10^{3}	Our Solution

5.3 Practice - Scientific Notation

Write each number in scientific notiation

1) 885

2) 0.000744

3) 0.081

4) 1.09

5) 0.039

6) 15000

Write each number in standard notation

7) $8.7 \text{ x } 10^5$

8) $2.56 \text{ x } 10^2$

9) $9 \text{ x } 10^{-4}$

10) $5 \text{ x } 10^4$

11) $2 \text{ x } 10^0$

12) $6 \text{ x } 10^{-5}$

Simplify. Write each answer in scientific notation.

13) $(7 \text{ x } 10^{-1})(2 \text{ x } 10^{-3})$

14) $(2 \times 10^{-6})(8.8 \times 10^{-5)}$

15) $(5.26 \text{ x } 10^{-5})(3.16 \text{ x } 10^{-2})$

16) $(5.1 \times 10^6)(9.84 \times 10^{-1})$

17) $(2.6 \text{ x } 10^{-2})(6 \text{ x } 10^{-2})$

18) $\frac{7.4 \times 10^4}{1.7 \times 10^{-4}}$

19) $\frac{4.9 \times 10^1}{2.7 \times 10^{-3}}$

20) $\frac{7.2 \times 10^{-1}}{7.32 \times 10^{-1}}$

21) $\frac{5.33 \times 10^{-6}}{9.62 \times 10^{-2}}$

22) $\frac{3.2 \times 10^{-3}}{5.02 \times 10^0}$

23) $(5.5 \times 10^{-5})^2$

24) $(9.6 \times 10^3)^{-4}$

25) $(7.8 \times 10^{-2})^5$

26) $(5.4 \times 10^6)^{-3}$

27) $(8.03 \times 10^4)^{-4}$

28) $(6.88 \times 10^{-4})(4.23 \times 10^1)$

29) $\frac{6.1 \times 10^{-6}}{5.1 \times 10^{-4}}$

30) $\frac{8.4 \times 10^5}{7 \times 10^{-2}}$

31) $(3.6 \times 10^0)(6.1 \times 10^{-3})$

32) $(3.15 \times 10^3)(8 \times 10^{-1})$

33) $(1.8 \times 10^{-5})^{-3}$

34) $\frac{9.58 \times 10^{-2}}{1.14 \times 10^{-3}}$

35) $\frac{9 \times 10^4}{7.83 \times 10^{-2}}$

36) $(8.3 \times 10^1)^5$

37) $\frac{3.22 \times 10^{-3}}{7 \times 10^{-6}}$

38) $\frac{5 \times 10^6}{6.69 \times 10^2}$

39) $\frac{2.4 \times 10^{-6}}{6.5 \times 10^0}$

40) $(9 \times 10^{-2})^{-3}$

41) $\frac{6 \times 10^3}{5.8 \times 10^{-3}}$

42) $(2 \times 10^4)(6 \times 10^1)$

5.3

Answers to Scientific Notation

1) 8.85×10^{2}

2) 7.44×10^{-4}

3) 8.1×10^{-2}

4) 1.09×10^{0}

5) 3.9×10^{-2}

6) 1.5×10^{4}

7) 870000

8) 256

9) 0.0009

10) 50000

11) 2

12) 0.00006

13) 1.4×10^{-3}

14) 1.76×10^{-10}

15) 1.662×10^{-6}

16) 5.018×10^{6}

17) 1.56×10^{-3}

18) 4.353×10^{8}

19) 1.815×10^{4}

20) 9.836×10^{-1}

21) 5.541×10^{-5}

22) 6.375×10^{-4}

23) 3.025×10^{-9}

24) 1.177×10^{-16}

25) 2.887×10^{-6}

26) 6.351×10^{-21}

27) 2.405×10^{-20}

28) 2.91×10^{-2}

29) 1.196×10^{-2}

30) 1.2×10^{7}

31) 2.196×10^{-2}

32) 2.52×10^{3}

33) 1.715×10^{14}

34) 8.404×10^{1}

35) 1.149×10^{6}

36) 3.939×10^{9}

37) 4.6×10^{2}

38) 7.474×10^{3}

39) 3.692×10^{-7}

40) 1.372×10^{3}

41) 1.034×10^{6}

42) 1.2×10^{6}